Collocated Sensors Bias Estimation in Autonomous Driving

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ABSTRACT

Sensors are prone to biases which can lead to inaccurate associations and hence poor results in target tracking. The sensors used on autonomous vehicles (AV) are placed together or very close (practically collocated) which makes the bias estimation challenging. This work considers the bias estimation for two collocated synchronized sensors with slowly varying additive biases. The biases' observability condition is met when the two sensors' biases are Ornstein-Uhlenbeck stochastic processes with different time constants. The proposed bias estimation is independent of state estimation and bias models are identified based on sample autocorrelations. With bias-compensated observations, the fused measurement can be obtained using the Maximum Likelihood fusion technique. In experiments, two collocated Lidars (different manufacturer models) are tested in real time. It is shown that the uncertainties of biases are significantly reduced by the estimation algorithm presented. The observation error reduction is up to 77% with bias-compensated measurement fusion and the bias uncertainty (root mean square error) has reduction up to 45% after fusion compared to the single Lidar scenario.

I. INTRODUCTION

Target tracking has always been an important problem for autonomous driving systems where multiple sensors are utilized to improve the tracking accuracy. Unfortunately, these sensors such as Radar, Lidar and cameras are prone to biases which can lead to problematic association and hence poor results in target tracking. The sensors used on autonomous driving vehicles can only be placed together or very close (practically collocated) which makes the bias estimation challenging. Consequently, only a few works even partially addressed this problem. Sensor calibration via off-line pre-processing is supposed to eliminate the sensor biases. However, it requires the knowledge of ground truth and to be of value the biases must be time-invariant. For the case where the sensor biases are dynamic and slowly varying, off-line calibration is not sufficient. This work proposes a real-time bias estimation method for collocated sensors independent from the target motion tracking and approaches for identifying the bias models.

The work [7] considered the bias estimation problem for radars on moving platforms by decoupling the tracking of targets of opportunity and the estimation of the radar and platform biases. Ref. [10] solved exact bias estimation for an active sensor by using pairs of range and angle measurements to create pseudo measurements of the biases of both sensors relying on the nonlinearity of the range and angle measurements. Ref. [3] considered time-varying bias estimation along with the target state. This work uses the differencing of the sensor measurements as in [10] to eliminate the target state in estimating the sensor biases, i.e., the biases can be estimated independently of the target. In [4], the authors considered the problem of estimating sensor biases from measurements of targets flying on known trajectories by augmenting the kinematic state vector with sensor bias parameters. In [10, 11], the bias model included scale biases and unknown locations of the sensors, as well as the usual offset biases. Ref. [8] considered 3-dimensional sensor bias estimation using sine-space measurements and showed the achievability of the Cramer-Rao lower bound.

To attack bias estimation for collocated sensors, this work considers slowly varying sensor biases that are modeled as Ornstein-Uhlenbeck (OU, a class of Gauss-Markov) processes (as discussed in [3]) and deals with

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bias estimation with the following contributions (i) solves the problem for collocated synchronized sensors; (ii) by using the difference between the associated sensor observations, the biases can be estimated independently from target motion; (iii) the proposed method can be applied to all kinds of observations (i.e., bearings, range, etc.) from various types of sensors.

The bias models (parameters of the OU process) are typically unknown with limited prior information. In [12, 13], the mean-reverting OU process parameter estimation is discussed along with long-term prediction. The approach introduced in [14] is used to identity the model parameters in numerical experiment. The prior information about the sensor biases consists of initial distributions, assumed to be Gaussian with zero mean and certain variances. The local observations can be fused, using the Maximum Likelihood criterion with bias compensated measurements, after the bias estimation. In experiments, two collocated Lidars (different manufacturer models) are tested in real time. It is shown that the estimation errors and uncertainties (root mean square error) of biases are significantly reduced by the estimation algorithm presented.

This paper is organized as follows. Sec. II formulates the problem by introducing the bias dynamic model, the bias measurement model (using the subtraction between the sensor observations) and discusses the bias observability. In Sec. III, the fusion of the bias-compensated observations is presented. The bias model identification method is presented in Sec. IV. Sec. V gives the numerical results. Conclusions and remarks are in Sec. VI.

II. PROBLEM FORMULATION

The sensor measurements from sensor i are

$$z_i(k) = h\left[\mathbf{x}(k), \mathbf{s}(k)\right] + b_i(k) + w_i(k) \qquad k = 1, 2, \ldots, N \tag{1}$$

where $\mathbf{x}$ is the true (common) target state, which is unknown, $\mathbf{s}$ is the sensor state* and $h[\cdot,\cdot]$ is the generic observation model (angle or range). The observation noise w_i is assumed to be Gaussian zero mean, white with variances $\sigma^2_{w_i}$ and the noises are independent of each other and from the sensor biases.

The challenge of this work is to estimate the (collocated) sensor biases efficiently given synchronized observations, which depend on both of the sensor and target motions as well as the biases and noises. It will be shown in the sequel that bias estimation is independent from the target state estimation, i.e., bias estimation and state tracking are decoupled. The observations considered are using generic models in one coordinate (i.e., with dimension of 1) to aid in the clarity of the exposition.

The sensor biases are slowly varying, modeled as an Ornstein–Uhlenbeck (first order Gauss-Markov) process [7]. The discrete dynamic model used for the biases [1, 3] is (two collocated sensors are considered in this work for simplicity),

$$b_i(k+1) = \alpha_i b_i(k) + v_i(k) \qquad i = 1, 2 \tag{2}$$

with

$$\alpha_i = e^{-T/\tau_i} \tag{3}$$

where T is the sampling interval and τ_i is the time constant of the bias evolution (assumed to be known; its estimation is discussed in Sec. III). The time constant is given in terms of α_i as

$$\tau_i = -T\,\text{ln}\alpha_i \tag{4}$$

The driving process noises v_i are assumed to be Gaussian zero mean, white with variances $\sigma^2_{v_i}$ independent across sensors. All the noises v_i and w_i are independent. Using the above model guarantees that the bias estimates are bounded since (2) is stable.

The steady-state mean-square (MS) value of the bias b_i is $\sigma^2_{b_i}$ and its relationship to the corresponding process noise variance is

$$\sigma^2_{v_i} = (1 - \alpha_i^2)\sigma^2_{b_i} \tag{5}$$

*Since the sensors are collocated, they share the same state.

To estimate the biases independently of the target state, only the difference between the observations is used

$$z(k) \triangleq z_1(k) - z_2(k) = b_1(k) - b_2(k) + w_1(k) - w_2(k) \tag{6}$$

and the bias state to be estimated is

$$\mathbf{b}(k) = [b_1(k)\ b_2(k)]' \tag{7}$$

with state equation

$$\mathbf{b}(k+1) = F\mathbf{b}(k) + \mathbf{v}(k) \tag{8}$$

where

$$F = \begin{bmatrix} \alpha_1 & 0 \\ 0 & \alpha_2 \end{bmatrix} \tag{9}$$

and the process noise vector is

$$\mathbf{v} = [v_1\ v_2]' \tag{10}$$

with covariance matrix

$$Q = \begin{bmatrix} \sigma_{v_1}^2 & 0 \\ 0 & \sigma_{v_2}^2 \end{bmatrix} \tag{11}$$

The measurement model based on (6) is

$$z(k) = H\mathbf{b}(k) + w(k) \tag{12}$$

where

$$H = \begin{bmatrix} 1 & -1 \end{bmatrix} \tag{13}$$

and the measurement noise $w(k)$ is

$$w(k) \triangleq w_1(k) - w_2(k) \tag{14}$$

which has variance $\sigma_{w_1}^2 + \sigma_{w_2}^2$.

A Kalman Filter (KF) can then be used for the estimation of $\mathbf{b}(k)$ which gives the estimate at time k [1]

$$\hat{\mathbf{b}}(k) = [\hat{b}_1(k)\ \hat{b}_2(k)]' \tag{15}$$

and the observability of the above system is discussed and verified via the observability matrix in [9,10].

It can be easily seen that the pair (F, C), where C is the Cholesky factor of the covariance matrix Q, is completely controllable. Also, the pair (F, H) is completely observable when $\alpha_1 \neq \alpha_2$. Thus, the solution of the discrete-time Riccati equation for such a time-invariant system will converge to a unique finite positive-definite steady-state covariance [1], which can obtained via KF.

III. FUSION OF THE OBSERVATIONS WITH BIAS COMPENSATION

Under the Gaussian assumption, the fusion of bias-compensated observations can be solved using the Maximum Likelihood (ML) criterion by maximizing the likelihood function (LF) or minimizing the negative log-likelihood function (NLLF) of the target position based on the observations from the two sensors.[†].

With the bias estimates, the current observation can be expressed as

$$z_i(k) = \zeta(k) + \hat{b}_i(k) + \tilde{b}_i(k) + w_i(k) \qquad i = 1, 2 \tag{16}$$

where

$$\zeta(k) = h\left[\mathbf{x}(k), \mathbf{s}(k)\right] \tag{17}$$

[†]The LF of a parameter of interest (in this case the target position) is the pdf of the observation conditioned on the parameter [1]

is the noiseless fused observation that needs to be estimated given $z_i(k), i = 1, 2$, the bias estimates $\hat{b}_i(k)$ and accounting for the residual bias error $\tilde{b}_i(k)$. The estimates for sensor biases are obtained using a Kalman Filter and next we estimate directly the fused observation with bias compensation.

The bias-compensated ("bc") observations, omitting the time argument k for simplicity, are

$$z_1^{\mathrm{bc}} = z_1 - \hat{b}_1 = \zeta + \tilde{b}_1 + w_1 \tag{18}$$

and

$$z_2^{\mathrm{bc}} = z_2 - \hat{b}_2 = \zeta + \tilde{b}_2 + w_2 \tag{19}$$

Under the ML criterion, the fusion is carried out by estimating ζ based on the bias-compensated observation vector $[z_1^{\mathrm{bc}}\ z_2^{\mathrm{bc}}]'$. The fused observation[‡] with bias compensation ("Fbc") is [1] Eq. (3.4.1-9)

$$\begin{aligned}\hat{\zeta}^{\mathrm{Fbc}} = &\left[(H^{\mathrm{Fbc}})'(R^{\mathrm{Fbc}})^{-1}H^{\mathrm{Fbc}}\right]^{-1} \\ &\cdot (H^{\mathrm{Fbc}})'(R^{\mathrm{Fbc}})^{-1}[z_1^{\mathrm{bc}}\ z_2^{\mathrm{bc}}]'\end{aligned} \tag{20}$$

where, in this case,

$$H^{\mathrm{Fbc}} = \begin{bmatrix}1\\1\end{bmatrix} \tag{21}$$

and

$$R^{\mathrm{Fbc}} = E\left\{\begin{bmatrix}\tilde{b}_1 + w_1\\ \tilde{b}_2 + w_2\end{bmatrix}\begin{bmatrix}\tilde{b}_1 + w_1 & \tilde{b}_2 + w_2\end{bmatrix}\right\} \tag{22}$$

$$= \begin{bmatrix}P_{11} + \sigma_{w_1}^2 & P_{12}\\ P_{12} & P_{22} + \sigma_{w_2}^2\end{bmatrix} \tag{23}$$

In (23), P_{mn} is the (m, n) element of the calculated covariance matrix associated with the bias estimate vector (15). The variance corresponding to the fused bias-compensated observation (20) is

$$P^{\mathrm{Fbc}} = \left[(H^{\mathrm{Fbc}})'(R^{\mathrm{Fbc}})^{-1}H^{\mathrm{Fbc}}\right]^{-1} \tag{24}$$

IV. MODEL PARAMETER IDENTIFICATION

The sensor biases are assumed to be Ornstein-Uhlenbeck (OU) processes as shown in (2). However, the model parameters are, in general, unknown but can be identified based on sample autocorrelation. The bias model identification is independent of and prior to the bias estimation. For simplicity, only one system is analyzed for illustration, however, the approach can be used for multiple sensors with the same model but different parameters.

Consider the discrete-time OU bias model (with slowly varying bias[§])

$$b(k+1) = \alpha b(k) + v(k) \tag{25}$$

and noisy observation model ($k = 1, 2, \ldots, N$)

$$z = h\left[\mathbf{x}(k), \mathbf{s}(k)\right] + b(k) + w(k) \tag{26}$$

$$o(k) \triangleq z(k) - h\left[\mathbf{x}(k), \mathbf{s}(k)\right] = b(k) + w(k) \tag{27}$$

with α defined in (3), v is the process noise with variance σ_v^2, and w is the measurement noise with variance σ_w^2; $\mathbf{x}$ and $\mathbf{s}$ are as in (1). The observation $o(k)$ is obtained assuming the truth is known, which can be done in the off-line pre-calibration. However, note that $h\left[\mathbf{x}(k), \mathbf{s}(k)\right]$ will cancel in (6), which will be used in estimating

‡The ML estimator is implemented using the LS technique [1].

§This implies that α is near unity and σ_v is small.

the two biases (15). The identification of the bias models will rely on (27). The bias model parameters can be estimated using sample autocorrelations.

The autocorrelation of $o(k)$, assumed to be wide-sense stationary (WSS), is

$$\begin{aligned} r(m) &= \mathrm{E}\{o(k)o(k-m)\} \\ &= \mathrm{E}\{[b(k)+w(k)][b(k-m)+w(k-m)]\} \\ &= \mathrm{E}\{b(k)b(k-m)\} + \sigma_x^2\delta(m) \\ &= r_b(m) + \sigma_x^2\delta(m) \end{aligned} \tag{28}$$

where $\delta(k)$ is the Kronecker delta function and $r_b(m)$ is the autocorrelation of $b(k)$, also assumed to be WSS,

$$\begin{aligned} r_b(m) &= \mathrm{E}\{b(k)b(k-m)\} \\ &= \mathrm{E}\{[\alpha b(k-1)+v(k-1)][b(k-m)]\} \\ &= \alpha^m r_b(0) \end{aligned} \tag{29}$$

and

$$\begin{aligned} r_b(0) &= \mathrm{E}\{b(k)b(k)\} \\ &= \mathrm{E}\{[\alpha b(k-1)+v(k-1)][\alpha b(k-1)+v(k-1)]^*\} \\ &= \alpha^2 r_b(0) + \sigma_v^2 \end{aligned} \tag{30}$$

The above yields

$$r_b(0) = \frac{\sigma_v^2}{1-\alpha^2} \tag{31}$$

Substituting (31) into (28) gives

$$r(m) = \alpha^m \frac{\sigma_v^2}{1-\alpha^2} + \sigma_w^2\delta(m) \tag{32}$$

which can be used to estimate α, σ_v^2 and σ_w^2. That is, assuming the sample autocorrelations are available (and sufficiently accurate since one can have only sample autocorrelations, i.e., time averages), one has

$$\alpha = \frac{r(2)}{r(1)} \tag{33}$$

$$\sigma_v^2 = \frac{r(1)^2 - r(2)^2}{r(2)} \tag{34}$$

$$\sigma_w^2 = r(0) - \frac{r(1)^2}{r(2)} \tag{35}$$

Note that the above solution is dependent on the accuracy of sample autocorrelations (especially when α is very close to 1), which can not be guaranteed with limited sample data. Assuming more autocorrelations (i.e., more than $r(2)$) are available, the parameters can be estimated using all sample autocorrelations to improve the accuracy [10].

V. NUMERICAL RESULTS

Numerical experiment results are given in this section. In the real-data experiment, two Lidars are placed together on top of the ego vehicle to detect a single object in a garage. The object (a stop sign) is static during the observation time and the true object position (ground truth) is measured manually. Lidar 1 is *Velodyne Alpha Prime* and Lidar 2 is *Velodyne HDL-32E*. The sensor observations are the bounding box centroid position, which is obtained using a separate processor, i.e., prior processing of Lidar point clouds for object detection[¶]. The scenario setup and detected object are is shown in Fig. 1 (marked in red). Note that, in the experiment,

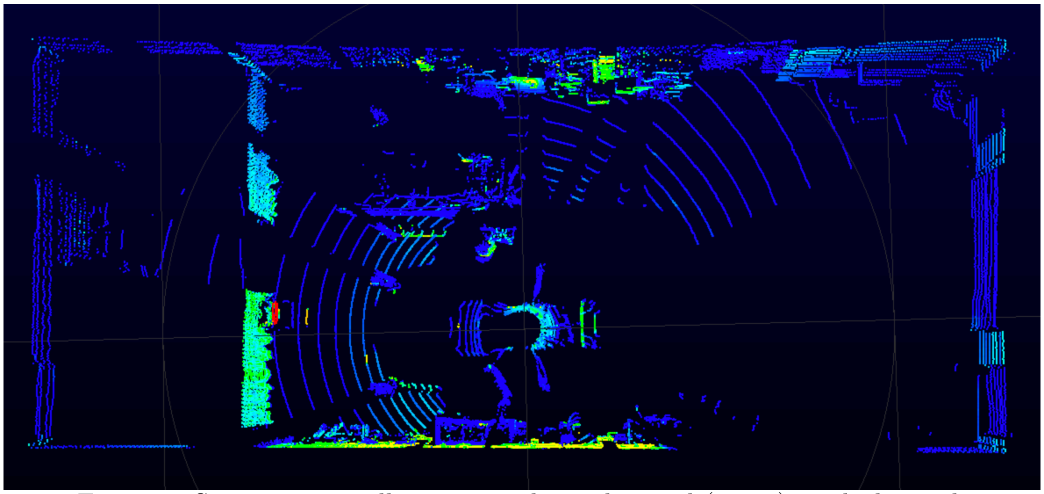

Figure 1: Scenario setup illustration: object detected (static) marked in red.

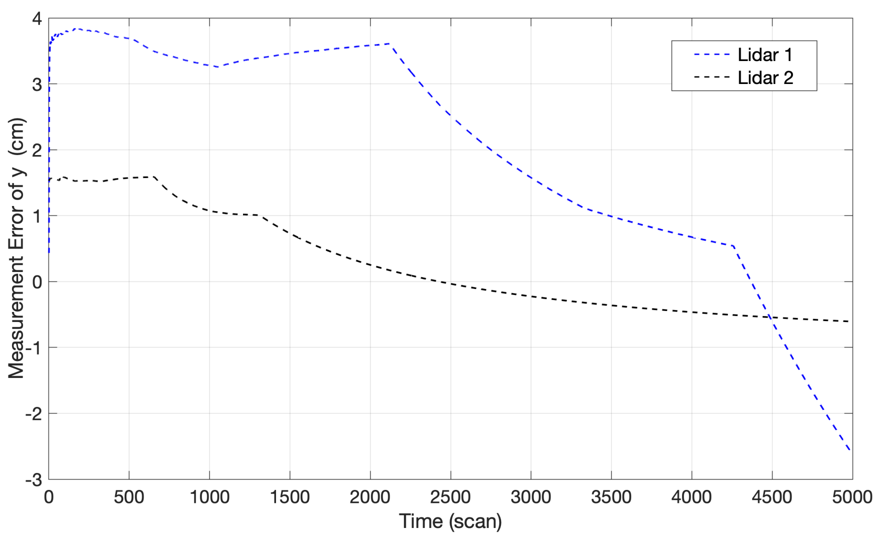

Figure 2: Measurement error (measurement minus ground truth) from both collocated sensors.

the object and sensors are aligned on x-axis, i.e., observations for x are 0 in a local coordinate system. Thus, the following results are only for the y-axis in local coordinates.

The sensors are assumed to be synchronized with a sampling rate of 10 Hz, i.e, both of the Lidars are providing position measurements in 2D (x, y) every 0.1 s. For bias system identification, 5000 scans of sample autocorrelations from each sensor are used to obtain the parameters such as time constant, process noise and measurement noise. The measurement errors (29) are shown in Fig. 2. The bias models are then used along with a Kalman filer for bias estimation using the method introduced in Sec II. Bias compensated fusion (under Least Squares criterion) is carried out for the last 500 scans for illustration. Fig. 3 shows the measurement error and fusion measurement error. It can be seen the error reduction is up to 77% for Lidar 1 and 52% for Lidar 2.

The fusion without bias compensated is considered to be a naïve fusion (with no bias compensation — "Fnbc") defined as

$$\hat{\zeta}^{\text{Fnbc}} = \frac{\sigma_{w_1}^{-2} z_1 + \sigma_{w_2}^{-2} z_2}{\sigma_{w_1}^{-2} + \sigma_{w_2}^{-2}} \tag{36}$$

¶This is beyond the scope of the present paper but can be found in the literature.

The corresponding variance is

$$P^{\mathrm{Fnbc}} = \frac{\sigma_{w_1}^{-4}\sigma_{w_1}^2 + \sigma_{w_2}^{-4}\sigma_{w_2}^2}{(\sigma_{w_1}^{-2} + \sigma_{w_2}^{-2})^2} \tag{37}$$

The result from naive fusion (36) is also presented in Fig. 3, it is clear that fusion with bias compensated has smaller error.

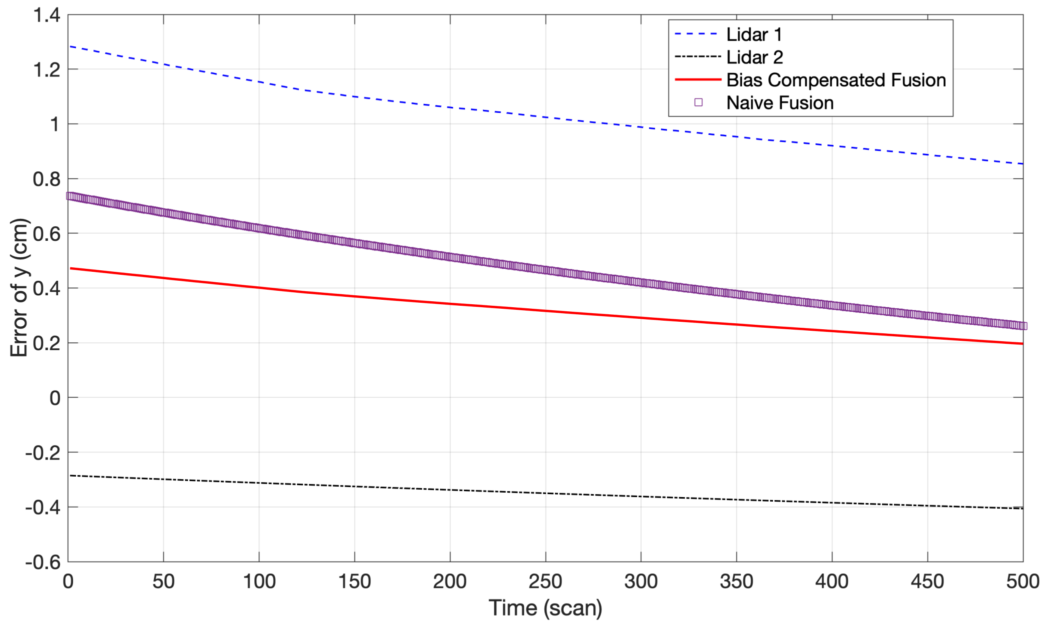

Figure 3: Observation error and fused measurement error with and without bias compensation.

We also analyze the uncertainty of the Lidar measurements and fused measurement. The root mean square errors (RMSE) of Lidar measurements and fused covariance (with and without bias compensation) are shown in Table 1. The reduction ratios (SD from fusion with bias compensation compared to others) are also listed. It can be seen that there are always improvements with bias compensated fusion.

Table 1: Uncertainty of Lidar measurements and fused measurement

Uncertainties	y (cm)	Reduction to Fusion w Bias Compensation $(\sigma - \sigma_{\mathrm{Fbc}})/\sigma$
Lidar 1 RMSE (σ_1)	1.14	45%
Lidar 2 RMSE (σ_2)	0.731	14%
SD of Naive Fusion (σ_{Fnbc})	0.728	13%
SD of bias-compensated Fusion (σ_{Fbc})	0.624	N/A

VI. CONCLUSIONS

In this work, the bias estimation for collocated synchronized sensors is solved using a target of opportunity. The sensor biases are slowly varying, modeled as Ornstein-Uhlenbeck processes. Only the difference between the sensor observations is used for the bias estimation, which is thus independent from the target state estimation. The system is observable when the biases have different eigenvalues in their noise-driven discrete time dynamic models. The bias model parameters are obtained directly via sample autocorrelations. With bias estimates and accounting for the residual biases, the fusion of the bias-compensated observations is carried out under the ML criterion. As shown in the numerical experiment, i.e., data collected from two Lidars, the fusion of the observations carried out with bias compensation results in significant reduction for the fused measurement. The estimation uncertainty will also improve with bias compensated fused measurements.

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